SUM 2019 MATH/STAT 394 Midterm

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Instructions

You have 1 hour to take the exam (DRS students excepted). You may use a single regular-sized sheet of notes, front and back. You may use a calculator.

1 Counting Marbles

You select marbles uniformly at random from a nonempty bag of $r \geq 0$ red marbles and $b \geq 0$ blue marbles.

(a) You draw 3 marbles **without replacement**. Find the probability that they are all three red marbles. You may assume that there are at least 3 marbles in the bag, but still be sure to consider the range of r and b and qualify your answer appropriately. (5 points if you find the solution using *either* an unordered *or* an ordered solution; 7 points if you do *both*.)

(b) Now you draw marbles **with replacement**, and stop when you draw a blue. Write down the sample space Ω **of strings of marble colors R and B** (rather than numbers of marbles, as we did in class). (2 points.)

(c) Continuing part (b), let the random variable X be the number of marble draws you make. Define X explicitly as a function of $\omega \in \Omega$. Also, what is the distribution (with parameter value) of X? (5 points.)

(d) Write down the p.m.f. of X, and show algebraically that it satisfies the 2 requirements a function must meet to be a p.m.f. Be sure to include any special cases of r, b. (8 points.)

2 Oomph Galoomph

There's a particularly vicious strain of the disease Galoomph (G) known as Oomph Galoomph (OG). You go to the doctor to get tested. The test has three outcomes: "-" (for negative/neither disease), "+" (for G), and "!" (for OG). Given that you are negative (N), there's an 80% chance you get a negative result, a 10% chance of "+," and a 10% chance of "!." (So, for example, $P(-|N) = 0.8$.) Given that you have G, there's an 80% chance of "+" and 10% chances of "-" and "!" each. However, if you have OG, there's just a 50% chance of "!" and a 50% chance of "-". 90% of the population is N, 9% has G, and 1% has OG.

(a) What's the probability that a randomly-chosen person will receive a diagnosis of "!" (OG)? You may round your answer to the hundredths place. (5 points.)

(b) What percentage of those randomly-selected individuals that are diagnosed with OG actually have it? You may use your rounded answer from the previous part and round again to the hundredths–hint hint! (10 points.)

(c) You receive a diagnosis of "!," so you wait until the effects of the previous test have left your system and get re-tested (so that the previous test doesn't interfere with the new test). Are the results of the two tests independent, conditionally independent (and if so, conditional on what?), or neither? Write a brief 1-2 sentence explanation. (5 points.)

(d) This second test also results in a "!" diagnosis. What is the probability that you have OG? If you did not bring a calculator, you need not do by-hand calculations for this part, but you must write a final, simplified expression in terms of variables and calculate each probability in that expression as a decimal/fraction of decimals. (5 points.)